

Thoughts on Non-Perturbative Thermalization and Jet Quenching in Heavy Ion Collisions

Yuri V. Kovchegov*

*Department of Physics, The Ohio State University
Columbus, OH 43210*

July, 2005

Abstract

We start by presenting physical arguments for the impossibility of perturbative thermalization leading to (non-viscous) Bjorken hydrodynamic description of heavy ion collisions. These arguments are complimentary to our more formal argument presented in [1]. We argue that the success of hydrodynamic models in describing the quark-gluon system produced in heavy ion collisions could only be due to non-perturbative strong coupling effects. We continue by studying non-perturbative effects in heavy ion collisions at high energies. We model non-perturbative phenomena by an instanton ensemble. We show that non-perturbative instanton vacuum fields may significantly contribute to jet quenching in nuclear collisions. At the same time, the instanton ensemble contribution to thermalization is likely to be rather weak, leading to non-perturbative thermalization time comparable to the time of hadronization. This example illustrates that jet quenching is not necessarily a signal of a thermalized medium. Indeed, since the instanton models do not capture all the effects of QCD vacuum (e.g. they do not account for confinement), there may be other non-perturbative effects facilitating thermalization of the system.

*e-mail: yuri@mps.ohio-state.edu

1 Introduction

The problem of thermalization has become one of the central questions in the theory of heavy ion collisions. The success of hydrodynamic models [2, 3, 4, 5, 6] in describing RHIC data on elliptic flow [7], along with the suppression of high- p_T particles in $Au+Au$ collisions as compared to $d+Au$ and pp collisions [8, 9, 10, 11, 12, 13, 14, 15, 17, 16, 18, 19, 20], indicate creation of a strongly interacting and likely thermal medium in the final state of heavy ion collisions — the quark-gluon plasma (QGP). Currently, the main theoretical puzzle is in understanding how this medium was formed in the collision.

The initial conditions for creation of QGP has received a lot of attention in the recent years [21, 22, 23, 24, 25, 26, 27] in the framework of saturation/Color Glass Condensate (CGC) physics [28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38]. The data on hadron multiplicity and spectra obtained in $d+Au$ collisions at RHIC [39, 40, 41, 42, 43] appears to be in good agreement with expectations of Color Glass approach [44, 45, 46, 47, 48, 49, 50, 51, 52, 53]. When combined with the success of saturation models [54] in describing total charged hadron multiplicity in $Au+Au$ collisions at RHIC, this allows one to hope that evolution of the produced quark-gluon system toward thermalization could also be described within the saturation/Color Glass framework, which indeed involves using Feynman diagrams.

However, understanding perturbative thermalization proved to be a very hard problem. Several perturbative thermalization scenarios have been proposed in the recent years [55, 56, 57, 58, 59, 60, 61]. None of them appears to lead to thermalization time as short as required by RHIC phenomenology, which is $\tau \lesssim 0.5$ fm/c. Moreover, the so-called "bottom-up" thermalization scenario, which was originally proposed in [55] and was the first one to use saturation initial conditions, proved to be susceptible to plasma instabilities [62, 63, 64, 65, 66, 67, 68, 69]. This brought hope that instability effects would speed up the onset of thermalization. However, in [65] it was shown that instabilities (along with any other dynamical effects) obey a lower bound on thermalization time, which is parametrically very close to (though slightly lower than) the thermalization time obtained in the "bottom-up" scenario [55]. More recent developments include suggestion to consider the onset of isotropization instead of thermalization as the necessary condition for hydrodynamic models to work [69], a numerical analysis of instabilities demonstrating that their magnitudes saturate as time progresses [70], leading to, probably, a less severe impact on the system as thought previously, and a new modified version of the "bottom-up" scenario [71].

In our recent paper on the subject [1] we considered the onset of thermalization as a possible transition from the $\epsilon \sim 1/\tau$ scaling of the energy density characteristic of the Color Glass condensate initial conditions [21, 22, 23, 24, 25, 26, 27], to a steeper falloff with proper time, $\epsilon \sim 1/\tau^{1+\Delta}$, expected if the system obeys ideal Bjorken hydrodynamics [2] ($\Delta > 0$). We were looking for perturbative corrections, which could be, for instance, powers of $\Delta \ln \tau$, with Δ being proportional to some power of the strong coupling constant α_s . Having found no such (or any other) important corrections to $1/\tau$ scaling of energy density we proved that no corrections of this type are possible in perturbation theory. Therefore, we proved that perturbative thermalization is impossible in heavy ion collisions. Our arguments in [1] were somewhat involved, and lacked a clear physical argument demonstrating impossibility of perturbative thermalization. Below we try to construct such an argument in Section 2. There we provide two arguments. The first one, presented in Sect. 2.1, demonstrates that, in the absence of non-perturbative

effects, if the theory is defined only perturbatively, Bjorken hydrodynamics would not lead to decoupling of the system. Therefore, plane-wave final states necessary in perturbation theory to define scattering amplitudes and particle production cross sections are never achieved. This indicates a contradiction between Bjorken hydrodynamics and perturbation theory, indicating possible impossibility of obtaining the former from the latter. The discrepancy becomes apparent in the second argument, presented in Sect. 2.2. There we argue that, in the zero coupling limit ($g \rightarrow 0$) taken at some late time after the collision, QCD perturbation theory would indeed lead to "free-streaming" with the energy density scaling like $\epsilon \sim 1/\tau$, while Bjorken hydrodynamics would give $\epsilon \sim 1/\tau^{4/3}$. The formal reason for such scaling in the case of Bjorken hydrodynamics is that the (non-interacting) ideal gas equation of state is $\epsilon = 3p$, and this is all that's needed to obtain the $\epsilon \sim 1/\tau^{4/3}$ scaling. The physical reason for $\epsilon \sim 1/\tau^{4/3}$ scaling in the $g \rightarrow 0$ limit is that, in writing $\epsilon = 3p$ equation of state for the ideal gas, one indeed assumes a presence of some thermal bath (e.g. the box containing the particles) with which the gas particles interact and which keeps them thermal throughout the evolution of the system. Such external thermal bath is absent in heavy ion collisions. Therefore, Bjorken hydrodynamics is impossible to achieve in the weak coupling limit. We further argue that another way to reach this conclusion is by noticing that shear viscosity, which is the first non-equilibrium correction to Bjorken hydrodynamics, diverges in the weak coupling limit (see Eq. (27)) [72], indicating that non-equilibrium corrections become comparable to the equilibrium values of the pressure, breaking down the ideal (non-viscous) hydrodynamics in the weak coupling limit. Our conclusion here is that thermalization of the quark-gluon system (if it does take place) leading to hydrodynamic description of the heavy ion collisions could only be due to non-perturbative strong coupling effects.

Indeed, understanding the non-perturbative phenomena in the non-equilibrium setting of heavy ion collisions is a very difficult (if not presently impossible) task. Here we will consider one well-studied model of non-perturbative effects — the instanton vacuum model [73, 74, 75, 76, 77, 78, 79]. Since, as we argue in Sect. 2, non-perturbative effects appear to be important in AA collisions for thermalization, which is required by phenomenology to describe the observed data on elliptic flow [4, 5, 7], we would like to investigate their influence on other important observables in heavy ion collisions. In Sect. 3 we study the effect of the instanton vacuum on jet quenching and thermalization in heavy ion collisions. Reviewing the results of [80] in Sect. 3.1 we argue that the instanton density is suppressed by strong perturbative gluon fields produced (in the Color Glass Condensate framework) in the early stages of the collision with the proper times $\tau \lesssim 1/Q_s$. At slightly later proper times, for $\tau \gtrsim 1/Q_s$, perturbative fields linearize and instanton vacuum is not suppressed anymore. At this point we would have a system of free quarks and gluons propagating through the instanton vacuum. Usually, a parton produced in a collision and propagating as a free plane wave would start hadronizing right away. This is probably true in pp or e^+e^- collisions. However, in AA collisions, at times $\tau \gtrsim 1/Q_s$, the energy density is still too high for hadronization to take place. Unable to hadronize due to high energy density, non-interacting quarks and gluons are likely to interact with the instantons in the vacuum and to loose energy as they propagate through the instanton ensemble until the hadronization time. (Once quarks and gluons are converted into hadrons they, of course, do not loose energy in the vacuum anymore.) The mechanism of parton-instanton interactions has been extensively studied in [81]. One might expect the energy loss of partons in the instanton vacuum to be negligibly small due to low density of instantons, which would play the role of

scattering centers. However, the peculiar property of the energy loss formula derived in [82, 83] and shown here in Eq. (47) is that the integrated partonic energy loss is particularly sensitive to late proper times of the collision due to an extra factor of τ in the integrand. Namely, it is not as important what the energy density was in the early stages of the collision, as it matters what the energy density is at late proper times. Therefore, the energy loss of a fast moving parton in a non-static medium is mostly determined by the medium density at late times. Since, at late proper times close to the time of hadronization, the density of scattering sources in the produced medium is comparable to the density of instantons in the vacuum, the latter may significantly contribute to jet quenching, on par with traditional energy loss in the produced medium, as we demonstrate by parametric estimates in Sect. 3.2.

At the same time, as the vacuum instanton density is low, its contribution to thermalization appears to be rather weak, leading to non-perturbative thermalization time comparable to the time of hadronization, as we show in Sect. 3.3. This example illustrates that jet quenching is not necessarily a signal of a thermalized medium and, if Eq. (47) adequately describes energy loss in a heavy ion collision, one may have significant jet quenching without thermalization. Of course, as we argue in Sect. 4, since the instanton models do not capture all the effects of QCD vacuum (e.g. they can not explain confinement), there may be other non-perturbative effects, such as QCD strings [84], facilitating thermalization of the system and leading to energy loss in the non-perturbatively formed QGP.

2 Non-perturbative Nature of Bjorken Hydrodynamics

In this Section we will present two qualitative arguments showing that perturbative thermalization leading to Bjorken hydrodynamics description of heavy ion collisions is impossible.

2.1 Argument I

Before we begin, let us briefly review the main assumptions of Bjorken hydrodynamics [2]. The main idea behind the ideal (non-viscous) Bjorken hydrodynamics is that the medium produced in a collision of two very large nuclei can be described by hydrodynamic equations

$$\partial_\mu T^{\mu\nu} = 0 \quad (1)$$

with the energy-momentum tensor given by

$$T_{\mu\nu} = (\epsilon + p) u_\mu u_\nu - p g_{\mu\nu}, \quad (2)$$

where, in the boost-invariant (rapidity-independent) case the fluid velocity is

$$u_\pm = \frac{x_\pm}{\tau}, \quad \underline{u} = 0 \quad (3)$$

and the proper time is $\tau = \sqrt{2x_+x_-}$. We use the light cone coordinates

$$x_\pm = \frac{t \pm z}{\sqrt{2}} \quad (4)$$

and \underline{u} denotes a two-dimensional transverse vector. ϵ is the energy density and p is the pressure of the fluid. Substituting Eq. (2) into Eq. (1) and assuming that $\epsilon = \epsilon(\tau)$ and $p = p(\tau)$ yields [2]

$$\frac{d\epsilon}{d\tau} = -\frac{\epsilon + p}{\tau}. \quad (5)$$

Eq. (5) leads to the scaling of energy density with the proper time [2]

$$\epsilon \sim \frac{1}{\tau^{1+\Delta}} \quad (6)$$

where

$$0 < \Delta \leq \frac{1}{3}. \quad (7)$$

The value of $\Delta = 1/3$ can be obtained in the ideal (ultrarelativistic) gas case with the equation of state

$$\epsilon = 3p \quad (8)$$

and leads to

$$\epsilon \sim \frac{1}{\tau^{4/3}}. \quad (9)$$

The value of $\Delta = 0$, leading to

$$\epsilon \sim \frac{1}{\tau} \quad (10)$$

could in principle be achieved in a QCD plasma near the critical phase transition temperature T_c if the phase transition was first order [2, 85]. However, the confining phase transition in QCD is non-perturbative: therefore it is impossible to obtain QCD matter at $T = T_c$ through Feynman diagrams and we will not be concerned with this case here.

Now, let us assume that, as a result of summation of perturbative diagrams to any order (and to all orders if necessary) one shows that in a heavy ion collision a thermalized medium is produced which is describable by Bjorken hydrodynamics. With logarithmic (due to running of the coupling) accuracy the energy density of the resulting medium would scale with the temperature T as

$$\epsilon \sim T^4. \quad (11)$$

This formula is better justified in the weak coupling limit, where the diagrammatic analysis is at its best too. Comparing Eq. (11) to Eq. (6) yields

$$T \sim \frac{1}{\tau^{\frac{1+\Delta}{4}}}. \quad (12)$$

For example, for ideal gas Eq. (12) yields $T \sim \tau^{-1/3}$.

Let us now imagine that thermalization and the subsequent evolution of the system are completely described by the perturbation theory. In other words, let us imagine “turning off” confinement and all other effects associated with the non-perturbative QCD scale Λ_{QCD} and assume that the theory is defined perturbatively. Indeed this is not a realistic assumption, but would help us clarify whether thermalization in heavy ion collisions is of perturbative nature or not: if thermalization is perturbative we should not get any inconsistencies by “turning off” confinement effects. If we do get inconsistencies, then perturbative thermalization would be

impossible, and if thermalization does take place in heavy ion collisions, it could only be due to non-perturbative effects.

For simplicity let us consider only gluons and massless quarks in the system. Since the temperature T is now the only scale in the problem, the perturbative mean free path of a (massless) quark or a gluon in the medium would scale like

$$\lambda_{pert} \sim \frac{1}{T}, \quad (13)$$

which, using Eq. (12) gives

$$\lambda_{pert} \sim \tau^{\frac{1+\Delta}{4}}. \quad (14)$$

Employing Eq. (7) we obtain an upper limit on the mean free path

$$\lambda_{pert} \sim \tau^{\frac{1+\Delta}{4}} \leq \tau^{1/3}. \quad (15)$$

The equality in Eq. (15) is achieved only for an ideal gas with the equation of state given by (8). (In Eq. (15) we have omitted τ -independent factors which would make the inequality dimensionally correct: putting them back into the formula would not modify the conclusion, but would unnecessarily complicate the equation. The same applies to Eq. (17) below.)

Noting that the typical longitudinal extent of the system at time t after a heavy ion collision is

$$L = t \geq \tau \quad (16)$$

we conclude that

$$\lambda_{pert} \leq \tau^{1/3} < \tau \leq L. \quad (17)$$

(The transverse extent of the system is assumed to be infinitely large [2] and is, therefore, always larger than the mean free path.)

Eq. (17) implies that the mean free path of a gluon or a massless quark in a plasma described by Bjorken hydrodynamics is always smaller than the size of the system. Therefore, the decoupling condition

$$\lambda \sim L \quad (18)$$

is *never achieved* in perturbative Bjorken plasma.

(Indeed, once we turn the confinement effects back on, we would get another scale in the problem, Λ_{QCD} , which would lead to

$$\lambda_{non-pert} = \frac{1}{\sigma \rho} \sim \frac{1}{\frac{1}{\Lambda_{QCD}^2} T^3} \sim \tau^{\frac{3(1+\Delta)}{4}} \quad (19)$$

giving the mean free path

$$\lambda_{non-pert} \sim \tau \sim L \quad (20)$$

for the ideal gas case ($\Delta = 1/3$), which marginally satisfies the decoupling condition. Here σ is the parton-parton scattering cross section and ρ is the density of matter.)

As we can see from Eq. (17), the asymptotic $\tau \rightarrow +\infty$ state of a perturbative plasma described by Bjorken hydrodynamics is not “free streaming”: in fact, the inequality in Eq. (17) gets more justified as τ increases. Each individual parton can escape the system with an exponentially small probability, $e^{-L/\lambda} \sim \exp(-\text{const } \tau^{2/3})$, which only decreases with time. This

behavior is in contradiction with the diagrammatic approach: the final state of any Feynman diagram in high energy scattering is a set of free non-interacting particles flying off to infinity. The existence of such a plane wave final state is an essential assumption of the perturbation theory. Without the plane wave final state the problem of particle production would be ill-defined. If diagrams would lead to Bjorken hydrodynamics, then, such a common observable as the one-gluon inclusive production cross section would be impossible to define and calculate in AA collisions, since there will be no plane-wave particles in the final state. Single gluon inclusive production cross section is well-defined in the quasi-classical approximation of McLerran-Venugopalan model [31, 21, 22, 23, 24, 25], which does not lead to a thermalized final state [24]. If thermalization comes in through the higher order quantum corrections to classical fields [55], such corrections would invalidate the notion of one-gluon inclusive production cross section, which appears absurd. Therefore, perturbative diagram resummation *can not* lead to Bjorken hydrodynamics: diagrams lead to a free streaming final state, while Bjorken hydrodynamics in the absence of confinement is an eternally interacting state.

Of course, for realistic finite nuclei, when the distance between the nuclei becomes comparable to their radii, the expansion becomes three-dimensional: in this sense Bjorken hydrodynamics is not really an “eternally” interacting state. The onset of 3D expansion would indicate a transition from Bjorken to Landau hydrodynamics and is outside of the scope of this discussion. It would be very unlikely though for thermalization to happen at this late stage of the collision when the system becomes very dilute and the energy density starts falling much faster with time than it was in the Bjorken hydrodynamics stage of the collision.

One may suggest that Feynman diagrams may lead to Bjorken hydrodynamics in some intermediate state preceding free streaming. Again this is not possible, since, as was shown above, Bjorken hydrodynamics does not lead to free streaming, and if the system enters a Bjorken hydrodynamics state it would never leave it. Therefore Bjorken hydrodynamics can not be an intermediate state of the system.

Introducing massive quarks in the argument is not going to change the conclusions: if the quark mass is much smaller than the temperature, $m_q \ll T$, then it could be neglected yielding $\sigma \sim \alpha_s^2/\alpha_s T^2 \sim 1/T^2$ and $\rho \sim T^3$ leading to the mean free path of Eq. (13). If quark mass is much bigger than the temperature, $m_q \gg T$, then the particle density would still be dominated by gluons yielding $\rho \sim T^3$ and the interaction cross sections would also be dominated by gluon exchanges, giving $\sigma \sim 1/T^2$, and we would recover Eq. (13) again.

The argument presented above shows that Bjorken hydrodynamics is fundamentally different, for instance, from the case of Landau hydrodynamics. In the latter case decoupling is possible even in the perturbative case. For simplicity let us consider an expanding three-dimensional spherical droplet with the energy density within the droplet independent of spacial coordinates and with the ideal gas equation of state (8). Then for such a system

$$\epsilon \sim \frac{1}{t^4}. \quad (21)$$

Following the above steps we obtain, using Eq. (11), that $T \sim 1/t$ and the mean free path is

$$\lambda_{pert} \sim \frac{1}{T} \sim t. \quad (22)$$

Since the radius of an ultrarelativistic droplet is $R \sim t$ we see that $\lambda_{pert} \sim R$ and the system may decouple at late times. Therefore, thermalization in this system, leading to Landau

hydrodynamics description of its evolution, appears to be possible to describe using Feynman diagrams as an intermediate state between the creation of the system and eventual decoupling. Indeed to make a definitive conclusion on the subject of perturbative thermalization in 3D expanding system one needs to perform a careful diagrammatic analysis similar to the one carried out in [1] for the case of Bjorken hydrodynamics.

One may worry that, for instance, Boltzmann equation has a diagrammatic interpretation [86], i.e., it does sum up certain classes of diagrams in its collision term, and, nevertheless, it does lead to perturbative thermalization. However we would like to point out that Boltzmann equation leads to thermalization for a system of finite volume, such as a gas of particles in a box. If a system is not confined to a fixed volume, the question of thermalization of the system in the weak coupling limit due to Boltzmann equation dynamics is still an open question which has to be addressed on individual basis for each system.

For the case of a system in a box, thermalization time in the weak coupling limit is comparable to the size of the box. In the diagrammatic language this implies that if we iterate the diagrams from the collision term of the Boltzmann equation to obtain a full resummed diagram describing the system, such diagram, covering the evolution of the system over a very long time scale in a fixed finite volume, should also include information on interactions of particles with the box in which they are confined, and, because of that, would not have an asymptotic “free streaming” final state. Therefore, it would not be a diagram in the vacuum and the above argument would not apply to it: one can think of the box as of an external field, which is present at all times preventing decoupling of particle interactions and not allowing them to free stream to spacial infinity.

To conclude this Section let us reiterate our argument once more. Bjorken hydrodynamics [2], in the weak coupling limit (in the absence of confinement effects), does not decouple: gluons and quarks continue interacting indefinitely, as long as one-dimensional expansion continues. Such system does not have a free final state, and, therefore can not be obtained from Feynman diagrams in vacuum, which, at any finite order in the coupling, and, therefore, to all orders as well, always do have a non-interacting final state. (Indeed Feynman diagrams in an external field, such as the graphs describing a system of particles in a box, do not have an asymptotically free final state: however, since, of course, there is no external field in heavy ion collisions, such diagrams would be irrelevant here.)

2.2 Argument II

Now let us assume that the system of quarks and gluons produced in a heavy ion collision reaches equilibration at some proper time τ_{th} . Let us pick some proper time τ_0 after thermalization, $\tau_0 \geq \tau_{th}$, and set the QCD coupling constant g to be equal to zero for all times after τ_0 , i.e., $g = 0$ for $\tau \geq \tau_0$. Below we will compare the $g \rightarrow 0$ limits of the full theory (QCD) and of Bjorken hydrodynamics.

First of all, from QCD standpoint, if the coupling goes to zero, $g \rightarrow 0$, the system would stop interacting and quarks and gluons would “free-stream” away. The energy density of a free-streaming quarks and gluons scales as

$$\epsilon \sim \frac{1}{\tau} \tag{23}$$

simply due to energy conservation in a one-dimensionally expanding system. (To obtain a different energy scaling with proper time, such as in Bjorken hydrodynamics case in Eq. (6), the system needs to perform work, which obviously does not happen with a free-streaming system.)

To analyze the $g \rightarrow 0$ limit of Bjorken hydrodynamics we first note that it requires only conservation of the energy-momentum tensor (Eqs. (1) and (2)) with an equation of state relating ϵ and p . Since, in QCD, perturbative equation of state is known and is given by [87]

$$\frac{\epsilon}{3p} = 1 + o(g^2), \quad (24)$$

we naturally conclude that in $g \rightarrow 0$ limit it reduces to the ideal gas equation of state (8). Using that equation of state in Eq. (5) governing Bjorken hydrodynamics [2] leads to

$$\epsilon \sim \frac{1}{\tau^{4/3}}. \quad (25)$$

Thus we have established that the $g \rightarrow 0$ limits of a field theory, such as QCD, and of Bjorken hydrodynamics are *different*, given by Eqs. (23) and (25) correspondingly. Therefore, perturbative QCD can never lead to ideal Bjorken hydrodynamics.

The origin of the difference between the two limits is clear: the ideal gas equation of state (8) describes a gas of particles which do not interact with each other, but which interact with some thermal bath. This thermal bath could be a box the particles are confined to or some external field. Indeed there is no external thermal bath in the case of a heavy ion collision: hence Bjorken hydrodynamic approximation has a different $g \rightarrow 0$ limit from the full theory.

One may worry that putting the coupling constant to zero at proper time $\tau \geq \tau_0$ would violate gauge invariance. Indeed this is true in general making our argument not quite rigorous. However, as was argued convincingly in the existing thermalization scenarios [55, 56, 57, 58, 59, 60, 61], by the time of possible thermalization (and even much earlier) the density of particles would become sufficiently low for the quarks and gluons to go on mass shell between the interactions. This would insure gauge invariance of the quark and gluon distributions at any given late proper time.

To reconcile the $g \rightarrow 0$ limit of Bjorken hydrodynamics from Eq. (25) with the more physically intuitive scaling of Eq. (23) one has to include *non-equilibrium* viscous corrections [88, 89]. Introducing shear viscosity dependence into the ideal Bjorken hydrodynamics energy-momentum tensor would make it depend on three variables — ϵ , p and η , and is accomplished by adding a viscosity-dependent term to Eq. (2). The modified energy-momentum tensor for a baryon-free fluid is [88, 72, 89]

$$T_{\mu\nu} = (\epsilon + p) u_\mu u_\nu - p g_{\mu\nu} + \eta \left(\nabla_\mu u_\nu + \nabla_\nu u_\mu - \frac{2}{3} \Delta_{\mu\nu} \nabla_\rho u^\rho \right), \quad (26)$$

where $\nabla_\mu \equiv (g_{\mu\nu} - u_\mu u_\nu) \partial^\nu$ and $\Delta_{\mu\nu} \equiv g_{\mu\nu} - u_\mu u_\nu$. The shear viscosity in perturbation theory for QCD with two light flavors is given by [72]

$$\eta = 86.473 \frac{T^3}{g^4 \ln(1/g)} \quad (27)$$

and is, therefore, *divergent* in the $g \rightarrow 0$ limit. Indeed this indicates importance of non-equilibrium corrections to Bjorken hydrodynamics, which was pointed out as a possibility already in the original paper [2]. The non-equilibrium corrections become infinitely large in the $g \rightarrow 0$ limit implying a possible breakdown of Bjorken hydrodynamics.

When viscous corrections are added to Bjorken hydrodynamics, the energy momentum tensor from Eq. (26) can be written as (in the local fluid rest frame) [88, 89]

$$T^{\mu\nu} = \begin{pmatrix} \epsilon(\tau) & 0 & 0 & 0 \\ 0 & p(\tau) + \frac{2}{3} \frac{\eta}{\tau} & 0 & 0 \\ 0 & 0 & p(\tau) + \frac{2}{3} \frac{\eta}{\tau} & 0 \\ 0 & 0 & 0 & p(\tau) - \frac{4}{3} \frac{\eta}{\tau} \end{pmatrix}. \quad (28)$$

As follows from Eq. (28), including viscosity-dependent additive term in the definition of $T_{\mu\nu}$ reduces the longitudinal pressure. This may lead to the energy density scaling as shown in Eq. (23), since the pressure p on the right hand side of Eq. (5) is, in fact, the longitudinal pressure component. In other words, substituting Eq. (28) into Eq. (1) modifies Eq. (5) to give [88]

$$\frac{d\epsilon}{d\tau} = -\frac{\epsilon + [p - (4/3) \eta/\tau]}{\tau}. \quad (29)$$

One can see that shear viscosity tends to reduce the expression in square brackets on the right hand side of Eq. (29), making the scaling of energy density ϵ with τ closer to that of Eq. (23). Indeed if shear viscosity is very large this expression in square brackets may become negative: however, large viscosity needed for that implies that other non-equilibrium corrections to the energy-momentum tensor of Eq. (26) (higher order derivatives of fluid velocity than the ones included in Eq. (26)) would also become important invalidating Eqs. (28) and (29). When departure from equilibrium becomes this large, it is not clear whether hydrodynamic approach would remain the best approximation to the relevant physics. Nevertheless, we have shown that non-equilibrium corrections tend to bring Bjorken hydrodynamics closer to the correct $g \rightarrow 0$ limit of the full theory, indicating that the system at weak coupling is *not* in equilibrium.

Here we would like to add a word of caution. As was shown in [1], the most general energy-momentum tensor for a central collision of two ultrarelativistic nuclei of infinite transverse extent is given by

$$T^{\mu\nu} = \begin{pmatrix} \epsilon(\tau) & 0 & 0 & 0 \\ 0 & p_2(\tau) & 0 & 0 \\ 0 & 0 & p_2(\tau) & 0 \\ 0 & 0 & 0 & p_3(\tau) \end{pmatrix} \quad (30)$$

at $z = 0$. Therefore the most general $T^{\mu\nu}$ depends only on three variables: energy density ϵ , and transverse (p_2) and longitudinal (p_3) pressure components. The form of the tensors in Eqs. (28) and (30) is very similar: in fact, one can always express p_2 and p_3 in terms of p and η and vice versa. Equal number of degrees of freedom in viscous hydrodynamics of Eq. (28) and in the most general energy-momentum tensor for the given collision geometry in Eq. (30) makes it harder to distinguish the system really obeying the rules of viscous hydrodynamics from the system in some non-equilibrium state.

Does our conclusion above indicate that ideal Bjorken hydrodynamics is impossible to achieve in heavy ion collisions? Possibly yes. However, one could also imagine the situation where the thermalization and hydrodynamic behavior of the quark-gluon system are due

to non-perturbative effects, which disappear in the $g \rightarrow 0$ limit. In that case hydrodynamic description would only be correct in the strong coupling limit of the theory. An example of possible scaling of energy density could be given by the following ansatz

$$\epsilon \sim \frac{1}{\tau^{1+\Delta} \exp(-c/\alpha_s)}, \quad (31)$$

with $\Delta > 0$ and $c > 0$ some non-perturbative constants. Indeed in the $\alpha_s \rightarrow 0$ limit Eq. (31) leads to Eq. (23). It would also explain the absence of perturbative $\alpha_s \ln \tau$ corrections to the scaling of Eq. (23) observed in [1]: the function in Eq. (31) is non-analytic in α_s and its Taylor series has zero coefficients for all non-zero powers of α_s . The strong coupling limit of Eq. (31), $\alpha_s \rightarrow \infty$, would give Eq. (6) thus recovering Bjorken hydrodynamics [2].

To conclude this Section let us summarize our argument presented here one more time: the difference of $g \rightarrow 0$ limits of perturbative QCD and Bjorken hydrodynamics, given by Eqs. (23) and (25) correspondingly, indicates that it is impossible for perturbative evolution of the system to lead to Bjorken hydrodynamics. Non-perturbative ansatze of the type shown in Eq. (31) are still possible though, indicating that success of hydrodynamic description of heavy ion collisions may truly be due to strong coupling effects, as appears to be demonstrated by RHIC phenomenology [4, 5, 89] and as suggested in some thermalization models [90].

3 Non-Perturbative Contributions to Jet Quenching and Thermalization

Below we will try to investigate the role of non-perturbative effects in heavy ion collisions by modeling them with an instanton ensemble. Instanton models have been very successful in describing many features of QCD vacuum. At the same time we are fully aware that instanton models do not capture *all* the properties of the QCD vacuum: the best-known example is the inability of instantons to explain confinement. Therefore, our discussion below describes only the contribution of instanton fields to the dynamics of heavy ion collisions, and does not include other non-perturbative (possibly strong coupling) effects, which may also be very important.

3.1 Instanton Density in Color Glass Background

The early stages of a heavy ion collision with proper times $\tau \lesssim 1/Q_s$, are dominated by strong (mostly classical) gluonic fields produced by the Color Glass Condensates in the colliding nuclei [31, 32, 33, 21, 22, 23, 24, 25]. As was shown in [80], these strong gluon fields modify the vacuum instanton density [77, 78]

$$n_0(\rho) = \frac{0.466 e^{-1.679 N_c}}{(N_c - 1)!(N_c - 2)!} \frac{1}{\rho^5} \left(\frac{2\pi}{\alpha_s(\rho)} \right)^{2N_c} e^{-\frac{2\pi}{\alpha_s(\rho)}}, \quad (32)$$

to give (see also [79])

$$n_{sat}(\rho) = n_0(\rho) \exp \left[\frac{\pi^3 \rho^4}{\alpha_s(N_c^2 - 1)} (A | G_{\mu\nu}^a(x) G_{\mu\nu}^a(x) - G_{\mu\nu}^a(x) \tilde{G}_{\mu\nu}^a(x) | A) \right]. \quad (33)$$

Here ρ is the instanton size, x is the instanton's position in space-time, $G_{\mu\nu}^a$ is the field strength tensor of the produced gluon field and $\langle A | \dots | A \rangle$ denotes averaging over nuclear wave functions. Eq. (33) shows that instanton density may be significantly affected by the strong gluonic fields produced in the collision.

The dominant gluon field produced in a heavy ion collision in the saturation/CGC framework is classical [21, 22, 23, 24, 25]. Since the complete classical gluon field is known only numerically [24], following [80] we will use the lowest order gluon field to calculate the correlators in Eq. (33). As was shown in [80], the lowest order ($\mathcal{O}(g^3)$) gluon field produced in a nuclear collision, which was calculated in [22], gives, in the forward light cone

$$\begin{aligned} \langle G_{\mu\nu}^a(x) G_{\mu\nu}^a(x) \rangle_{LO} &= -\frac{g^6}{(2\pi)^2} \frac{A_1 A_2}{S_1 S_2} \frac{C_F}{\pi^3} \int \frac{d^2 k}{(\underline{k}^2)^2} \\ &\times \frac{dk_+ dk_- dk'_+ dk'_-}{(2\pi)^2} \frac{e^{-ik_+ x_- - ik_- x_+ - ik'_+ x_- - ik'_- x_+}}{(k^2 + i\epsilon k_0)(k'^2 + i\epsilon k'_0)} k \cdot k', \end{aligned} \quad (34)$$

where A_1 and A_2 are atomic numbers of two nuclei which, for simplicity, are taken to be cylinders with their axes aligned with the collision axis and with cross-sectional areas S_1 and S_2 correspondingly. In arriving at Eq. (34) one has to use the following formula (see Appendix B of [1], n, m are integers)

$$\int_{-\infty}^{\infty} dk_+ dk_- \frac{e^{-ik_+ x_- - ik_- x_+}}{k^2 + i\epsilon k_0} k_+^n k_-^m = -2\pi^2 \left(\frac{i k_T \tau}{2 x_-} \right)^n \left(\frac{-i k_T \tau}{2 x_+} \right)^m J_{m-n}(k_T \tau) \quad (35)$$

to replace each product $k_+ k_-$ and $k'_+ k'_-$ by $k_T^2/2$ in anticipation of the integration over k_+, k_-, k'_+, k'_- . (If $n = m$ the power of $(k_+ k_-)^n$ on the left hand side of Eq. (35) only brings in a power of $(k_T^2/2)^n$ on its right hand side.) Here $k_T \equiv |\underline{k}|$ and in Eq. (34) $\underline{k}' = -\underline{k}$ [80].

With the help of Eq. (35) we can perform the integration over k_+, k_-, k'_+, k'_- in Eq. (34) obtaining

$$\langle G_{\mu\nu}^a(x) G_{\mu\nu}^a(x) \rangle_{LO} = -16 \alpha_s^3 C_F \frac{A_1 A_2}{S_1 S_2} \int \frac{d^2 k}{k_T^4} k_T^2 \{ [J_0(k_T \tau)]^2 - [J_1(k_T \tau)]^2 \}. \quad (36)$$

Using the fact that, in the same lowest order approximation, the multiplicity of the produced gluons with transverse momentum k_T , rapidity y and located at impact parameter \underline{b} is [21, 22, 23]

$$\frac{dN}{d^2 k dy d^2 b} = \frac{8 \alpha_s^3 C_F}{\pi} \frac{A_1 A_2}{S_1 S_2} \frac{1}{k_T^4} \ln \frac{k_T}{\Lambda}, \quad (37)$$

and neglecting the logarithm, we rewrite Eq. (36) approximately as

$$\langle G_{\mu\nu}^a(x) G_{\mu\nu}^a(x) \rangle_{LO} \approx -2\pi \int d^2 k \frac{dN}{d^2 k dy d^2 b} k_T^2 \{ [J_0(k_T \tau)]^2 - [J_1(k_T \tau)]^2 \}. \quad (38)$$

At the same time, as was shown in [80]

$$\langle G_{\mu\nu}^a(x) \tilde{G}_{\mu\nu}^a(x) \rangle_{LO} = 0. \quad (39)$$

Combining Eqs. (38) and (39) with Eq. (33) we write

$$n_{sat}(\rho) = n_0(\rho) \exp \left\{ -\frac{\pi^3 \rho^4}{\alpha_s (N_c^2 - 1)} 2\pi \int d^2k \frac{dN}{d^2k dy d^2b} k_T^2 \left\{ [J_0(k_T \tau)]^2 - [J_1(k_T \tau)]^2 \right\} \right\}. \quad (40)$$

To analyze Eq. (40) we will use the fact that the gluon transverse momentum spectrum due to classical fields scales as $\sim 1/k_T^4$ for $k_T \gtrsim Q_s$, as shown in Eq. (37), and is approximately constant (up to logarithms) for $k_T \lesssim Q_s$ [21, 22, 23, 24, 25]. This implies that the k_T -integral in Eq. (40) is (approximately) dominated by $k_T \approx Q_s$. At early proper times, for $\tau \lesssim 1/Q_s$, the Bessel function $J_1(k_T \tau) \approx k_T \tau/2$ can be neglected compared to $J_0(k_T \tau) \approx 1$ in Eq. (40) leading to instanton suppression as was pointed out in [80].

On the other hand, at late times, when $\tau \gtrsim 1/Q_s$, we use the large-argument asymptotics of Bessel function to write

$$[J_0(k_T \tau)]^2 - [J_1(k_T \tau)]^2 \approx \frac{2}{\pi k_T \tau} \cos \left(2 k_T \tau - \frac{\pi}{2} \right), \quad k_T \tau \gg 1. \quad (41)$$

At these late times the exponent of Eq. (40) starts oscillating around 0 with a decreasing amplitude. This means that instanton suppression by produced gluon fields disappears for $\tau \gtrsim 1/Q_s$.

This disappearance of suppression has a straightforward physical meaning. As was shown in [1], any (classical and/or quantum) gluon fields in ultrarelativistic heavy ion collisions lead to the dominant contribution to the resulting energy density scaling as

$$\epsilon \approx \frac{\pi}{2} \int d^2k \frac{dN}{d^2k d\eta d^2b} k_T^2 \left\{ [J_1(k_T \tau)]^2 + [J_0(k_T \tau)]^2 \right\}. \quad (42)$$

At proper times $\tau \gtrsim 1/Q_s$, Eq. (42) leads to the energy scaling of Eq. (23), corresponding to free streaming [1]. This means that by the proper time $\tau \gtrsim 1/Q_s$ the gluon fields would linearize and propagate as plane waves. Plane wave gluons have equal chromo-electric and chromo-magnetic fields, $E^a = B^a$. Remembering that $G_{\mu\nu}^{a2} = 2(\vec{B}^{a2} - \vec{E}^{a2})$ we conclude that $G_{\mu\nu}^{a2} \approx 0$ at times $\tau \gtrsim 1/Q_s$ and suppression of instanton density disappears from Eq. (33).

3.2 Instanton Contribution to Jet Quenching

Let us summarize the emerging picture of the collision. At early times, $\tau \lesssim 1/Q_s$, strong gluon fields are produced and instantons are suppressed. At later, but still comparatively early times, $\tau \gtrsim 1/Q_s$, the gluon fields linearize and vacuum effects due to instantons are not suppressed anymore. The energy density of the linearized gluon fields is still quite high at $\tau \approx 1/Q_s$, since, for $\tau \geq 1/Q_s$

$$\epsilon \sim \frac{Q_s^3}{\alpha_s \tau}, \quad (43)$$

as follows from Eq. (42). (The factor of $1/\alpha_s$ in Eq. (43) is due to the dominance of classical gluon fields at the early stages of the collision [21, 22, 23, 24, 25, 26, 27].) From here on we will only perform parametric estimates to demonstrate the principle points: detailed calculations will be left for future analyses.

We assume that hadronization does not set in until the energy density of the gluon medium becomes of the order of $\epsilon_h \sim \Lambda_{QCD}^4$. This energy density is obtained by requiring that we have one hadron with mass $\sim \Lambda_{QCD}$ per unit volume in the rest frame of the hadron, which is roughly $\sim 1/\Lambda_{QCD}^3$. Equating the energy density from Eq. (43) to Λ_{QCD}^4 yields

$$\tau_h \sim \frac{Q_s^3}{\alpha_s \Lambda_{QCD}^4}. \quad (44)$$

Eq. (44) assumes that one-dimensional expansion lasts indefinitely long. Of course in a realistic heavy ion collision one dimensional expansion lasts only up to $\tau \sim R$ with R the nuclear radius. After that the expansion becomes three-dimensional and the hadronization energy density $\epsilon_h \sim \Lambda_{QCD}^4$ is achieved faster: we will return to this below.

As we saw in Sect. 3.1, at early times, $\tau \lesssim 1/Q_s$, the non-perturbative instanton effects are suppressed and the vacuum is perturbative. At later times, $\tau \gtrsim 1/Q_s$, instantons are not suppressed anymore and the vacuum is back to being non-perturbative. Deconfined gluons and quarks propagating through this vacuum would interact with the gluon (and quark) fields of the instantons. This may lead to energy loss by these partons in the instanton ensemble. Quarks and gluons, which, in a purely perturbative scenario, would have been propagating as plane waves for proper times $1/Q_s \lesssim \tau \leq \tau_h$, start interacting with the non-perturbative vacuum fields losing their energy. Again this phenomenon is much more pronounced in AA collisions, since in this case gluons and quarks spend a much longer time in the deconfined state interacting with the vacuum fields than they would in pp , pA or e^+e^- collisions. Indeed after hadronization, when the gluons and quarks would become confined within hadrons, the energy loss in vacuum fields would stop: hadrons are eigenstates of the QCD hamiltonian and they do not lose energy in propagation.

The idea of partonic energy loss in the strong non-perturbative fields has been previously put forward by Shuryak and Zahed in [91]. However, in their analysis the non-perturbative fields (sphalerons) were generated in a nuclear collision (see also [92]), and were not the QCD vacuum fields.

To estimate the energy loss in the instanton vacuum fields by a propagating parton we will employ the results of calculations carried out in [15, 16, 17, 18, 19, 20, 82, 83]. Following [82] we write the following expression for the energy loss of a quark created in a medium and having traveled distance z in that medium

$$-\frac{dE}{dz} = \frac{\alpha_s N_c}{2} \frac{1}{2-\alpha} z \rho(z) \int d^2q q_T^2 \frac{d\sigma}{d^2q}, \quad (45)$$

where $\rho(z)$ is the density of the medium at point z . (Gluon energy loss is different from Eq. (45) only by a Casimir.) Here α is a constant depending on the rate of the medium expansion with time, $0 < \alpha < 1$ [82]. Since we are interested in parametric estimates only, the exact value of α is not important to us. For the same reason the exact form of the (integrated) scattering cross section on a color charge in the medium in Eq. (45) is not important: we will just use the fact that it is

$$\int d^2q q_T^2 \frac{d\sigma}{d^2q} \sim \alpha_s^2. \quad (46)$$

Using such parametric estimates, we conclude that the integrated energy loss of a quark or gluon produced in a medium at proper time τ_0 which travels through the medium up to some

hadronization time τ_h is

$$-\Delta E \sim \alpha_s^3 \int_{\tau_0}^{\tau_h} d\tau \tau \rho(\tau). \quad (47)$$

Below we will use Eq. (47) to estimate the energy loss of a hard parton in the instanton vacuum fields and in the produced medium.

Of course the energy density of the instanton vacuum is much lower (at early times) than the energy density of the produced medium. Therefore, one might assume the energy loss in vacuum fields to be negligibly small compared to the energy loss in the produced medium. However, for a wide range of functions $\rho(\tau)$, the τ -integral in Eq. (47) is dominated by *late* proper times τ due to an extra factor of τ in the integrand. Therefore, for the integrated energy loss, the density of the medium at late times is more important than the density of the medium at earlier times. Since, at late times, the density of produced medium becomes comparable to the instanton density in vacuum, the contributions of both the medium and the vacuum to partonic energy loss are quite comparable, as we will see below.

The instanton density in the four-dimensional space-time is approximately [78]

$$n \approx 1 \text{ fm}^{-4} \approx \Lambda_{QCD}^4. \quad (48)$$

Assuming that each instanton (or anti-instanton) provides at least one scattering center for the energy loss calculations of the type carried out in [15, 16, 17, 18, 19, 20, 82, 83], Eq. (48) allows us to put a lower bound on the resulting density of scattering centers at

$$\rho_{inst}(\tau) \approx \Lambda_{QCD}^3. \quad (49)$$

Substituting Eq. (49) into Eq. (47) yields the following energy loss in the instanton vacuum fields ($\tau_h \gg \tau_0$)

$$-\Delta E_{inst} \sim \alpha_s^3 \Lambda_{QCD}^3 \tau_h^2. \quad (50)$$

To see whether the energy loss given by our estimates in Eq. (50) is large or small we want to compare it to the energy loss in the quark-gluon plasma. Indeed, as we argued above, this thermalized medium (QGP) can not be generated perturbatively. Below we will investigate whether parton-instanton interactions can lead to creation of a thermalized medium at early times. In the meantime we would just assume that a thermal medium with the energy density scaling given by Eq. (9) has been created in the collision. For this medium, requiring that the energy density should map onto Eq. (43) at $\tau = 1/Q_s$, we write

$$\epsilon_{qgp} \sim \frac{Q_s^4}{\alpha_s (Q_s \tau)^{4/3}}. \quad (51)$$

Remembering that if $\epsilon \sim T^4$ then $\rho \sim T^3$ we deduce from Eq. (51) the density of the produced medium

$$\rho_{qgp}(\tau) \sim \frac{Q_s^3}{\alpha_s^{3/4} Q_s \tau}. \quad (52)$$

Substituting Eq. (52) into Eq. (47) gives us a parametric estimate of energy loss for a parton in quark-gluon plasma

$$-\Delta E_{qgp} \sim \alpha_s^{9/4} Q_s^2 \tau_h. \quad (53)$$

Now we can compare the energy loss in the instanton vacuum from Eq. (50) to the energy loss in QGP from Eq. (53). First, assuming that nuclei have infinite transverse extent, such that the hadronization time is given by Eq. (44), we substitute it into Eqs. (50) and (53) to obtain

$$-\Delta E_{inst} \sim \alpha_s \frac{Q_s^6}{\Lambda_{QCD}^5} \quad (54)$$

and

$$-\Delta E_{qgp} \sim \alpha_s^{5/4} \frac{Q_s^5}{\Lambda_{QCD}^4}. \quad (55)$$

Since $Q_s \gg \Lambda_{QCD}$ and $\alpha_s \ll 1$ we conclude that

$$|\Delta E_{inst}| \gg |\Delta E_{qgp}|, \quad (56)$$

i.e., that the energy loss in the instanton vacuum is actually somewhat greater than the energy loss in the produced medium! Since the saturation scale grows with energy and atomic number, $Q_s^2 \sim A^{1/3} s^{\lambda/2}$, the instanton-induced energy loss would increase with energy and the system's size faster than the energy loss in a medium.

However, the conclusion of Eq. (56) was reached for the one-dimensional expansion only. For realistic nuclei one-dimensional expansion stops at $\tau \approx R$. Since the hadronization time τ_h from Eq. (44) is parametrically much larger than R , the one-dimensional expansion stops before the time τ_h is achieved. After that the expansion becomes three-dimensional. Assuming that free streaming continues in the 3D expansion, the energy density for the proper times after $\tau \approx R$ is given by

$$\epsilon = \epsilon(\tau = R) \left(\frac{R}{\tau} \right)^3 \quad (57)$$

where $\epsilon(\tau = R)$ could be obtained by putting $\tau = R$ in Eq. (43). Eq. (57) becomes

$$\epsilon \sim \frac{Q_s^3}{\alpha_s R} \left(\frac{R}{\tau} \right)^3. \quad (58)$$

Equating the energy density from Eq. (58) to $\epsilon_h \sim \Lambda_{QCD}^4$ we obtain the hadronization time for a realistic nuclear collision

$$\tau_h^{3D} \sim R \frac{Q_s}{\Lambda_{QCD}} \frac{1}{(\alpha_s R \Lambda_{QCD})^{1/3}}. \quad (59)$$

If we use the hadronization time from Eq. (59) in Eqs. (50) and (53) we get

$$-\Delta E_{inst} \sim \alpha_s^{7/3} \frac{Q_s^2}{\Lambda_{QCD}} (R \Lambda_{QCD})^{4/3} \quad (60)$$

and

$$-\Delta E_{qgp} \sim \alpha_s^{23/12} \frac{Q_s^3}{\Lambda_{QCD}^2} (R \Lambda_{QCD})^{2/3}. \quad (61)$$

To compare Eqs. (60) and (61) we first analyze them in the quasi-classical approximation: this regime seems to be adequate for mid-rapidity AA collisions at RHIC. As in the quasi-classical

McLerran-Venugopalan model $Q_s^2 \approx A^{1/3} \Lambda_{QCD}^2$ and since $R \Lambda_{QCD} \sim A^{1/3}$, we can conclude, neglecting the factors of α_s in the front, that for Eqs. (60) and (61)

$$|\Delta E_{inst}| \sim A^{7/9} > |\Delta E_{qgp}| \sim A^{13/18}. \quad (62)$$

Again, the instanton-induced energy loss appears to be slightly higher than the one in the plasma. Indeed, when quantum evolution corrections are included the saturation scale gets extra enhancement as $Q_s^2 \sim s^{\lambda/2}$: in that case $|\Delta E_{qgp}|$ from Eq. (61) would grow faster with energy than $|\Delta E_{inst}|$ from Eq. (60), eventually becoming larger. Still we have shown that the energy loss in the instanton vacuum is parametrically comparable to the energy loss in QGP even for realistic nuclear collisions, except for very high energies when $Q_s \gg \Lambda_{QCD} (R \Lambda_{QCD})^{2/3}$. The instanton energy loss from both Eqs. (54) and (60) also increases with energy since $Q_s^2 \sim s^{\lambda/2}$, which is in qualitative agreement with the absence of suppression of hadron production at lower (SPS) energy AA collisions and with the onset of suppression at higher (RHIC) energies.

Indeed there is no real physical competition between the energy loss in QGP and in the instanton vacuum. If QGP is formed, instantons would be suppressed [93], much like suppression in CGC that was discussed in Sect. 3.1. Here we calculate energy loss in QGP just to have a reference to compare instanton energy loss to. We need to do this since all our estimates in this Section are parametric and can not be used to produce numbers to compare to experimental data.

Let us also note that a significant energy loss due to instanton fields only takes place in AA collisions. For instance, in pA collisions energy loss is given by Eqs. (60) and (61) with the nuclear radius R replaced by the proton radius, $\tau_h^p \sim r_p \sim 1/\Lambda_{QCD}$ and the nuclear saturation scale Q_s replaced by the proton saturation scale, which, in the quasi-classical approximation is roughly $\sim \Lambda_{QCD}$. After performing these substitutions we get

$$-\Delta E_{inst}^{pA} \sim \Lambda_{QCD} \sim -\Delta E_{qgp}^{pA}, \quad (63)$$

which is very small, much smaller than the energy loss in AA collisions given by Eqs. (60) and (61).

To summarize, we have succeeded in showing that the energy loss in the vacuum fields modeled by an instanton ensemble is appreciably high, being parametrically comparable to the energy loss expected in hot quark-gluon plasma. Once again, let us reiterate that such conclusion is only possible due to a peculiar form of the integrated energy loss in Eq. (47), which was derived in [82, 83], making ΔE mostly sensitive to the color charge density at late proper times. The obtained instanton energy loss increases with both center of mass energy of the collisions and with the size of the system.

3.3 Instanton Contribution to Thermalization

Finally, let us address the following question: if the interactions of the quarks and gluons with the instanton vacuum fields are sufficiently strong to generate a significant energy loss, as shown in Eqs. (54) and (60), should not the same interactions lead to thermalization of the quark-gluon medium? This is a difficult question which, in general, requires understanding to what extent interactions of partons with the instanton fields would trigger interactions of partons with each other, which, in a purely perturbative approach [24, 1], would stop by $\tau \approx 1/Q_s$.

Here we will postpone this important question for further investigation, which is beyond the scope of this work, and concentrate on parton-instanton interactions only.

As was shown in [24, 1] the perturbative gluon fields are produced by $\tau \approx 1/Q_s$ with zero longitudinal pressure component. Namely, at $z = 0$ and at $\tau \approx 1/Q_s$ the energy-momentum tensor looks (approximately) like $T_{\mu\nu} = \text{diag}(\epsilon, p, p, 0)$. Thermalization of the system must be accompanied by generation of the non-zero longitudinal pressure component comparable to the transverse pressure components [69]. This is a necessary, though not sufficient, condition for thermalization. Let us estimate when such a longitudinal pressure component would be developed in parton-instanton interactions: that would give us a lower bound on thermalization time.

Avoiding all the subtleties of parton-instanton interactions [81] we assume that the upper bound on the cross section of a parton-instanton interaction is given by the instanton size squared, $\sigma_{inst} \sim \langle \rho \rangle^2 \sim 1/\Lambda_{QCD}^2$ [78]. The mean free path of a parton in the instanton medium with the density given by Eq. (49) is then

$$\lambda_{inst} = \frac{1}{\rho_{inst} \sigma_{inst}} \sim 1/\Lambda_{QCD}. \quad (64)$$

In each parton-instanton interaction, the momentum transfer to the parton would be of the order or $1/\langle \rho \rangle \sim \Lambda_{QCD}$. After N rescatterings the parton's longitudinal momentum gets broadened to approximately

$$p_z \sim \sqrt{N} \Lambda_{QCD}. \quad (65)$$

For the system to become isotropic we demand that $p_z \sim p_T \sim Q_s$, which gives $N \sim Q_s^2/\Lambda_{QCD}^2$. The time it takes to have $N \sim Q_s^2/\Lambda_{QCD}^2$ parton-instanton rescatterings is roughly given by

$$\tau_{th} \sim \lambda_{inst} N \sim \frac{Q_s^2}{\Lambda_{QCD}^3}. \quad (66)$$

This is a very late thermalization time. Even in McLerran-Venugopalan model, where $Q_s^2 \approx A^{1/3} \Lambda_{QCD}^2$, this thermalization time becomes comparable to the nuclear radius, $\tau_{th} \sim R$, which corresponds to the end of the one-dimensional expansion. Certainly such thermalization time is too late for hydrodynamic models describing RHIC data [4, 5]. Including energy dependence in Q_s would only make τ_{th} larger, leading to thermalization time which increases with energy. Indeed a numerical prefactor in Eq. (66), when calculated, may change the actual numerical results somewhat. Nevertheless we have demonstrated that, at least for very large nuclei, instanton vacuum can significantly contribute to jet quenching, as was shown in Eq. (60), but is not likely to bring in fast thermalization.

4 Conclusions

Above we have presented two arguments demonstrating that perturbative thermalization leading to ideal Bjorken hydrodynamics is impossible in heavy ion collisions. The only possible way to reconcile this with the phenomenological evidence for fast equilibration of the quark-gluon system produced in the collision [4, 5] is to suggest that thermalization takes place through some non-perturbative processes. It might be reasonable to assume that non-perturbative effects become important at proper times $\tau_\Lambda \approx 1/\Lambda_{QCD}$ in AA collisions, as they are known

to do in pp , pA and e^+e^- collisions. Non-perturbative effects involve QCD strings [84] with the string tension $\sigma \approx 1$ GeV/fm. Gluons and quarks produced in the initial stages of the collision carry a typical transverse momentum of the order of Q_s , which is $Q_s = 1 \div 1.4$ GeV at RHIC [54]. This number is comparable to $\sigma \tau_\Lambda \approx 1$ GeV, indicating that QCD string effects may become important at proper time τ_Λ , introducing strong-coupling interactions between the partons and, possibly, bringing the system towards fast thermalization. The corresponding time scale $\tau_\Lambda \approx 1/\Lambda_{QCD} \approx 1$ fm/c appears to be consistent with the thermalization time of approximately 0.5 fm/c required by hydrodynamical models [4, 5] to describe RHIC elliptic flow data [7].

Non-perturbative thermalization, either through instantons or through QCD string interactions, would not violate our argument of impossibility of perturbative thermalization presented above and in [1]. As we have argued in Sect. 2.2, thermalization in heavy ion collisions could only be a strong coupling phenomenon: such effects lie outside of the analysis carried out in [1]. The small coupling α_s used in the energy loss discussion of Sect. 3.2 is due to parton momentum broadening in the non-perturbative instanton vacuum and does not infringe on the non-perturbative nature of the vacuum. Indeed, the thermalization scenario discussed in Sect. 3.3 is essentially partonic, but it requires non-perturbative background instanton fields, which are also outside of the perturbative analysis of [1].

What appears to be unclear in non-perturbative thermalization scenarios with either instanton or string interactions is the energy dependence of thermalization time. It is not clear how string-induced interactions "know" about the center-of-mass energy of the collision. Instanton-mediated thermalization appears to have thermalization time (66) increasing with energy. This contradicts the fact that thermalization time required by phenomenology at RHIC is short, while it is not clear whether thermalization has ever occurred at the SPS implying a much longer thermalization time at lower energies.

Non-perturbative effects appear to be needed to describe jet quenching in heavy ion collisions as well. As we discussed above in Sect. 3.1, in the purely perturbative scenario of the collision, by the proper time $\tau \sim 1/Q_s$ the gluon fields would linearize and propagate as plane waves. (Similar results have been obtained for quark fields.) This result implies that, as long as perturbative interactions are concerned, nothing happens to the gluons after $\tau \sim 1/Q_s$. Therefore, the resulting gluon spectra would be completely determined by the (very) early stage dynamics. This would be in contradiction to the observation of Cronin enhancement at mid-rapidity in $d + Au$ collisions at RHIC [94, 40] and the observation of suppression of hadron production at mid-rapidity in $Au + Au$ collisions [95]: all the known perturbative production mechanisms imply that, since hadrons produced at mid-rapidity in $d + Au$ collisions exhibit Cronin enhancement, so should the partons in the early stages of $Au + Au$ collisions. Therefore, to reconcile RHIC data with the theory one has to assume that strong final state interactions take place in AA collisions, which could only be non-perturbative as was demonstrated by the above analysis. Such effects could be due to non-perturbative thermalization leading to formation of QGP in which the partons would loose energy. Alternatively, as we have shown above, partons can loose energy in the non-perturbative vacuum fields, to which they are exposed until the hadronization time in heavy ion collisions.

To summarize we note that we have demonstrated above that interactions in the final stages of heavy ion collisions can only be non-perturbative in nature. They may or may not lead to rapid thermalization, but they are likely to significantly contribute to jet quenching.

Acknowledgments

The author would like to thank Peter Arnold, Eric Braaten, and Ulrich Heinz for many informative discussions.

This work is supported in part by the U.S. Department of Energy under Grant No. DE-FG02-05ER41377.

References

- [1] Y. V. Kovchegov, arXiv:hep-ph/0503038.
- [2] J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).
- [3] K. J. Eskola, K. Kajantie and P. V. Ruuskanen, Eur. Phys. J. C **1**, 627 (1998) [arXiv:nucl-th/9705015].
- [4] P. F. Kolb, U. W. Heinz, P. Huovinen, K. J. Eskola and K. Tuominen, Nucl. Phys. A **696**, 197 (2001) [arXiv:hep-ph/0103234]; P. Huovinen, P. F. Kolb, U. W. Heinz, P. V. Ruuskanen and S. A. Voloshin, Phys. Lett. B **503**, 58 (2001) [arXiv:hep-ph/0101136]; P. F. Kolb, P. Huovinen, U. W. Heinz and H. Heiselberg, Phys. Lett. B **500**, 232 (2001) [arXiv:hep-ph/0012137]; P. F. Kolb, J. Sollfrank and U. W. Heinz, Phys. Rev. C **62**, 054909 (2000) [arXiv:hep-ph/0006129].
- [5] D. Teaney, J. Lauret and E. V. Shuryak, arXiv:nucl-th/0110037; Nucl. Phys. A **698**, 479 (2002) [arXiv:nucl-th/0104041]; Phys. Rev. Lett. **86**, 4783 (2001) [arXiv:nucl-th/0011058]; D. Teaney and E. V. Shuryak, Phys. Rev. Lett. **83**, 4951 (1999) [arXiv:nucl-th/9904006].
- [6] T. Hirano and Y. Nara, Nucl. Phys. A **743**, 305 (2004) [arXiv:nucl-th/0404039].
- [7] K. H. Ackermann *et al.* [STAR Collaboration], Phys. Rev. Lett. **86**, 402 (2001) [arXiv:nucl-ex/0009011]; S. S. Adler *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **91**, 182301 (2003) [arXiv:nucl-ex/0305013]; B. B. Back *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **89**, 222301 (2002) [arXiv:nucl-ex/0205021].
- [8] S. S. Adler [PHENIX Collaboration], arXiv:nucl-ex/0306021.
- [9] B. B. Back [PHOBOS Collaboration], arXiv:nucl-ex/0306025.
- [10] J. Adams [STAR Collaboration], arXiv:nucl-ex/0306024.
- [11] I. Arsene *et al.* [BRAHMS Collaboration], Phys. Rev. Lett. **91**, 072305 (2003) [arXiv:nucl-ex/0307003].
- [12] K. Adcox *et al.* [PHENIX Collaboration], Phys. Lett. B **561**, 82 (2003) [arXiv:nucl-ex/0207009]; S. S. Adler *et al.* [PHENIX Collaboration], arXiv:nucl-ex/0304022.
- [13] B. B. Back *et al.* [PHOBOS Collaboration], arXiv:nucl-ex/0302015.

- [14] J. Adams *et al.* [STAR Collaboration], arXiv:nucl-ex/0305015; C. Adler *et al.* [STAR Collaboration], Phys. Rev. Lett. **89**, 202301 (2002) [arXiv:nucl-ex/0206011].
- [15] J. D. Bjorken, FERMILAB-PUB-82-059-THY (unpublished).
- [16] R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne and D. Schiff, Nucl. Phys. B **483**, 291 (1997) [arXiv:hep-ph/9607355]; Nucl. Phys. B **484**, 265 (1997) [arXiv:hep-ph/9608322]; Nucl. Phys. B **478**, 577 (1996) [arXiv:hep-ph/9604327]; R. Baier, Y. L. Dokshitzer, A. H. Mueller and D. Schiff, Nucl. Phys. B **531**, 403 (1998) [arXiv:hep-ph/9804212]; JHEP **0109**, 033 (2001) [arXiv:hep-ph/0106347]; R. Baier, Y. L. Dokshitzer, S. Peigne and D. Schiff, Phys. Lett. B **345**, 277 (1995) [arXiv:hep-ph/9411409].
- [17] X. N. Wang, M. Gyulassy and M. Plumer, Phys. Rev. D **51**, 3436 (1995) [arXiv:hep-ph/9408344]; M. Gyulassy, P. Levai and I. Vitev, Phys. Rev. Lett. **85**, 5535 (2000) [arXiv:nucl-th/0005032]; M. Gyulassy, I. Vitev and X. N. Wang, Phys. Rev. Lett. **86**, 2537 (2001) [arXiv:nucl-th/0012092]; I. Vitev and M. Gyulassy, Phys. Rev. Lett. **89**, 252301 (2002) [arXiv:hep-ph/0209161]; X. N. Wang, arXiv:nucl-th/0305010; E. Wang and X.-N. Wang, Phys. Rev. Lett. **89**, 162301 (2002).
- [18] A. Kovner and U. A. Wiedemann, arXiv:hep-ph/0304151 and references therein.
- [19] B.G. Zakharov, JETP Lett. **63**, 952 (1996); **65**, 615 (1997).
- [20] C.A. Salgado and U.A. Wiedemann, Phys. Rev. Lett. **89**, 092303 (2002); Phys. Rev. D **68**, 014008 (2003) [arXiv:hep-ph/0302184].
- [21] A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. D **52**, 3809 (1995) [arXiv:hep-ph/9505320]; Phys. Rev. D **52**, 6231 (1995) [arXiv:hep-ph/9502289].
- [22] Y. V. Kovchegov and D. H. Rischke, Phys. Rev. C **56**, 1084 (1997) [arXiv:hep-ph/9704201].
- [23] M. Gyulassy and L. D. McLerran, Phys. Rev. C **56**, 2219 (1997) [arXiv:nucl-th/9704034].
- [24] A. Krasnitz and R. Venugopalan, Nucl. Phys. B **557**, 237 (1999) [arXiv:hep-ph/9809433]; Phys. Rev. Lett. **84**, 4309 (2000) [arXiv:hep-ph/9909203]; Phys. Rev. Lett. **86**, 1717 (2001) [arXiv:hep-ph/0007108]; A. Krasnitz, Y. Nara and R. Venugopalan, Phys. Rev. Lett. **87**, 192302 (2001) [arXiv:hep-ph/0108092].
- [25] Y. V. Kovchegov, Nucl. Phys. A **692**, 557 (2001) [arXiv:hep-ph/0011252]; Nucl. Phys. A **698**, 619 (2002) [arXiv:hep-ph/0106313].
- [26] A. Krasnitz, Y. Nara and R. Venugopalan, Nucl. Phys. A **717**, 268 (2003) [arXiv:hep-ph/0209269].
- [27] T. Lappi, Phys. Rev. C **67**, 054903 (2003) [arXiv:hep-ph/0303076]; Phys. Rev. C **70**, 054905 (2004) [arXiv:hep-ph/0409328].
- [28] L. V. Gribov, E. M. Levin and M. G. Ryskin, Phys. Rept. **100**, 1 (1983).
- [29] A. H. Mueller and J. w. Qiu, Nucl. Phys. B **268**, 427 (1986).

- [30] J. P. Blaizot and A. H. Mueller, Nucl. Phys. B **289**, 847 (1987).
- [31] L. D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994) [arXiv:hep-ph/9309289]; Phys. Rev. D **49**, 3352 (1994) [arXiv:hep-ph/9311205]; Phys. Rev. D **50**, 2225 (1994) [arXiv:hep-ph/9402335].
- [32] Y. V. Kovchegov, Phys. Rev. D **54**, 5463 (1996) [arXiv:hep-ph/9605446]; Phys. Rev. D **55**, 5445 (1997) [arXiv:hep-ph/9701229].
- [33] J. Jalilian-Marian, A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. D **55**, 5414 (1997) [arXiv:hep-ph/9606337].
- [34] A. H. Mueller, Nucl. Phys. B **415**, 373 (1994); A. H. Mueller and B. Patel, Nucl. Phys. B **425**, 471 (1994) [arXiv:hep-ph/9403256]; A. H. Mueller, Nucl. Phys. B **437**, 107 (1995) [arXiv:hep-ph/9408245]; Z. Chen and A. H. Mueller, Nucl. Phys. B **451**, 579 (1995).
- [35] I. Balitsky, Nucl. Phys. B **463**, 99 (1996) [arXiv:hep-ph/9509348]; Y. V. Kovchegov, Phys. Rev. D **60**, 034008 (1999) [arXiv:hep-ph/9901281]; Phys. Rev. D **61**, 074018 (2000) [arXiv:hep-ph/9905214].
- [36] J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, Nucl. Phys. B **504**, 415 (1997) [arXiv:hep-ph/9701284]; Phys. Rev. D **59**, 014014 (1999) [arXiv:hep-ph/9706377]; Phys. Rev. D **59**, 034007 (1999) [Erratum-ibid. D **59**, 099903 (1999)] [arXiv:hep-ph/9807462]; J. Jalilian-Marian, A. Kovner and H. Weigert, Phys. Rev. D **59**, 014015 (1999) [arXiv:hep-ph/9709432]; E. Iancu, A. Leonidov and L. D. McLerran, Phys. Lett. B **510**, 133 (2001) [arXiv:hep-ph/0102009]; E. Iancu and L. D. McLerran, Phys. Lett. B **510**, 145 (2001) [arXiv:hep-ph/0103032]; E. Ferreira, E. Iancu, A. Leonidov and L. McLerran, Nucl. Phys. A **703**, 489 (2002) [arXiv:hep-ph/0109115]; E. Iancu, K. Itakura and L. McLerran, Nucl. Phys. A **708**, 327 (2002) [arXiv:hep-ph/0203137]; for a review see E. Iancu and R. Venugopalan, arXiv:hep-ph/0303204.
- [37] M. Gyulassy and L. McLerran, Nucl. Phys. A **750**, 30 (2005) [arXiv:nucl-th/0405013].
- [38] J. Jalilian-Marian and Y. V. Kovchegov, arXiv:hep-ph/0505052.
- [39] R. Debbe, [BRAHMS Collaboration], talk given at the APS DNP Meeting at Tucson, AZ, October, 2003; R. Debbe [BRAHMS Collaboration], arXiv:nucl-ex/0403052.
- [40] I. Arsene *et al.* [BRAHMS Collaboration], Phys. Rev. Lett. **93**, 242303 (2004) [arXiv:nucl-ex/0403005].
- [41] S. S. Adler *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **94**, 082302 (2005) [arXiv:nucl-ex/0411054].
- [42] B. B. Back *et al.* [PHOBOS Collaboration], Phys. Rev. C **70**, 061901 (2004) [arXiv:nucl-ex/0406017].

- [43] G. Rakness [STAR Collaboration], talks given at XXXXth Rencontres de Moriond, “QCD and Hadronic Interactions at High Energy”, March 12-19, 2005, La Thuile, Italy and at the XIII International Workshop on Deep Inelastic Scattering (DIS2005), April 27 - May 1, 2005, Madison, Wisconsin, USA; see also L. S. Barnby [STAR Collaboration], [arXiv:nucl-ex/0404027].
- [44] D. Kharzeev, E. Levin and M. Nardi, Nucl. Phys. A **730**, 448 (2004) [Erratum-ibid. A **743**, 329 (2004)] [arXiv:hep-ph/0212316].
- [45] D. Kharzeev, E. Levin and L. McLerran, Phys. Lett. B **561**, 93 (2003) [arXiv:hep-ph/0210332].
- [46] D. Kharzeev, Y. V. Kovchegov and K. Tuchin, Phys. Rev. D **68**, 094013 (2003) [arXiv:hep-ph/0307037]; Phys. Lett. B **599**, 23 (2004) [arXiv:hep-ph/0405045].
- [47] J. L. Albacete, N. Armesto, A. Kovner, C. A. Salgado and U. A. Wiedemann, Phys. Rev. Lett. **92**, 082001 (2004) [arXiv:hep-ph/0307179].
- [48] R. Baier, A. Kovner and U. A. Wiedemann, Phys. Rev. D **68**, 054009 (2003) [arXiv:hep-ph/0305265].
- [49] J. Jalilian-Marian, Y. Nara and R. Venugopalan, Phys. Lett. B **577**, 54 (2003) [arXiv:nucl-th/0307022].
- [50] Yu. V. Kovchegov and A. H. Mueller, Nucl. Phys. B **529**, 451 (1998) [arXiv:hep-ph/9802440].
- [51] A. Dumitru and L. D. McLerran, Nucl. Phys. A **700**, 492 (2002) [arXiv:hep-ph/0105268].
- [52] Yu. V. Kovchegov and K. Tuchin, Phys. Rev. D **65**, 074026 (2002) [arXiv:hep-ph/0111362].
- [53] M. A. Braun, Phys. Lett. B **483**, 105 (2000) [arXiv:hep-ph/0003003].
- [54] D. Kharzeev and M. Nardi, Phys. Lett. B **507**, 121 (2001) [arXiv:nucl-th/0012025]; D. Kharzeev, E. Levin and M. Nardi, arXiv:hep-ph/0111315; Nucl. Phys. A **730**, 448 (2004) [Erratum-ibid. A **743**, 329 (2004)] [arXiv:hep-ph/0212316]; D. Kharzeev and E. Levin, Phys. Lett. B **523**, 79 (2001) [arXiv:nucl-th/0108006].
- [55] R. Baier, A.H. Mueller, D. Schiff and D.T. Son, Phys. Lett. **B502**, 51 (2001); Phys. Lett. **B539**, 46 (2002).
- [56] A. H. Mueller, Phys. Lett. B **475**, 220 (2000) [arXiv:hep-ph/9909388]; Nucl. Phys. B **572**, 227 (2000) [arXiv:hep-ph/9906322].
- [57] J. Serreau and D. Schiff, J. High Energy Phys. **0111**, 039 (2001).
- [58] J. Bjorker and R. Venugopalan, Phys. Rev. C **63**, 024609 (2001).
- [59] A. Dumitru and M. Gyulassy, Phys. Lett. **B494**, 215 (2000).
- [60] D. Molnar and M. Gyulassy, Nucl. Phys. **A697**, 495 (2002).

- [61] S. M. H. Wong, arXiv:hep-ph/0404222. Nucl. Phys. A **607**, 442 (1996) [arXiv:hep-ph/9606305]; Phys. Rev. C **54**, 2588 (1996) [arXiv:hep-ph/9609287]; Phys. Rev. C **56**, 1075 (1997) [arXiv:hep-ph/9706348].
- [62] U. W. Heinz, Nucl. Phys. A **418** (1984) 603C.
- [63] S. Mrowczynski, Phys. Lett. B **214**, 587 (1988); **314**, 118 (1993); **393**, 26 (1997) [arXiv:hep-ph/9606442]; Phys. Rev. C **49**, 2191 (1994); Phys. Rev. D **65**, 117501 (2002) [arXiv:hep-ph/0112100]; S. Mrowczynski and M. H. Thoma, Phys. Rev. D **62**, 036011 (2000) [arXiv:hep-ph/0001164].
- [64] P. Arnold, J. Lenaghan and G. D. Moore, JHEP **0308**, 002 (2003) [arXiv:hep-ph/0307325].
- [65] P. Arnold and J. Lenaghan, Phys. Rev. D **70**, 114007 (2004) [arXiv:hep-ph/0408052].
- [66] J. Randrup and S. Mrowczynski, Phys. Rev. C **68**, 034909 (2003) [arXiv:nucl-th/0303021].
- [67] P. Romatschke and M. Strickland, Phys. Rev. D **68**, 036004 (2003) [arXiv:hep-ph/0304092]; arXiv:hep-ph/0406188.
- [68] S. G. Matinyan, B. Muller and D. H. Rischke, Phys. Rev. C **57**, 1927 (1998) [arXiv:nucl-th/9708053].
- [69] P. Arnold, J. Lenaghan, G. D. Moore and L. G. Yaffe, Phys. Rev. Lett. **94**, 072302 (2005) [arXiv:nucl-th/0409068]; A. Rebhan, P. Romatschke and M. Strickland, Phys. Rev. Lett. **94**, 102303 (2005) [arXiv:hep-ph/0412016].
- [70] P. Arnold, G. D. Moore and L. G. Yaffe, arXiv:hep-ph/0505212.
- [71] A. H. Mueller, A. I. Shoshi and S. M. H. Wong, arXiv:hep-ph/0505164.
- [72] P. Arnold, G. D. Moore and L. G. Yaffe, JHEP **0011**, 001 (2000) [arXiv:hep-ph/0010177]; G. Baym, H. Monien, C. J. Pethick and D. G. Ravenhall, Phys. Rev. Lett. **64**, 1867 (1990).
- [73] A. A. Belavin, A. M. Polyakov, A. S. Shvarts and Y. S. Tyupkin, Phys. Lett. B **59**, 85 (1975); G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976).
- [74] I. I. Balitsky and A. V. Yung, Phys. Lett. B **168**, 113 (1986).
- [75] D. Diakonov and V. Y. Petrov, Nucl. Phys. B **272**, 457 (1986).
- [76] A. Ringwald and F. Schrempp, Phys. Lett. B **459**, 249 (1999) [arXiv:hep-lat/9903039].
- [77] G. 't Hooft, Phys. Rev. D **14**, 3432 (1976) [Erratum-ibid. D **18**, 2199 (1978)]; C. W. Bernard, N. H. Christ, A. H. Guth and E. J. Weinberg, Phys. Rev. D **16**, 2967 (1977); L. S. Brown and D. B. Creamer, Phys. Rev. D **18**, 3695 (1978); A. I. Vainshtein, V. I. Zakharov, V. A. Novikov and M. A. Shifman, Sov. Phys. Usp. **24**, 195 (1982) [Usp. Fiz. Nauk **136**, 553 (1982)].
- [78] T. Schafer and E. V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998) [arXiv:hep-ph/9610451].

- [79] C. G. Callan, R. F. Dashen and D. J. Gross, Phys. Rev. D **19**, 1826 (1979); M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **165**, 45 (1980).
- [80] D. E. Kharzeev, Y. V. Kovchegov and E. Levin, Nucl. Phys. A **699**, 745 (2002) [arXiv:hep-ph/0106248].
- [81] A. Ringwald, Nucl. Phys. B **330**, 1 (1990); O. Espinosa, Nucl. Phys. B **343**, 310 (1990); L. D. McLerran, A. I. Vainshtein and M. B. Voloshin, Phys. Rev. D **42**, 171 (1990); I. I. Balitsky and V. M. Braun, Phys. Lett. B **314**, 237 (1993) [arXiv:hep-ph/9305269]; Phys. Rev. D **47**, 1879 (1993); Nucl. Phys. B **380**, 51 (1992); V. A. Rubakov, D. T. Son and P. G. Tinyakov, Phys. Lett. B **287**, 342 (1992); V. I. Zakharov, Nucl. Phys. B **383**, 218 (1992); D. E. Kharzeev, Y. V. Kovchegov and E. Levin, Nucl. Phys. A **690**, 621 (2001) [arXiv:hep-ph/0007182]; M. A. Nowak, E. V. Shuryak and I. Zahed, Phys. Rev. D **64**, 034008 (2001) [arXiv:hep-ph/0012232].
- [82] R. Baier, Y. L. Dokshitzer, A. H. Mueller and D. Schiff, Phys. Rev. C **58**, 1706 (1998) [arXiv:hep-ph/9803473].
- [83] M. Gyulassy, I. Vitev, X. N. Wang and B. W. Zhang, arXiv:nucl-th/0302077; M. Gyulassy, I. Vitev, X. N. Wang and P. Huovinen, Phys. Lett. B **526**, 301 (2002) [arXiv:nucl-th/0109063].
- [84] N. Armesto, M. A. Braun, E. G. Ferreira and C. Pajares, Phys. Rev. Lett. **77**, 3736 (1996) [arXiv:hep-ph/9607239]; A. Capella, E. G. Ferreira and A. B. Kaidalov, Eur. Phys. J. C **11**, 163 (1999) [arXiv:hep-ph/9903338].
- [85] F. Karsch and E. Laermann, arXiv:hep-lat/0305025.
- [86] A. H. Mueller and D. T. Son, Phys. Lett. B **582**, 279 (2004) [arXiv:hep-ph/0212198]; S. Jeon, arXiv:hep-ph/0412121; S. p. Li and L. D. McLerran, Nucl. Phys. B **214**, 417 (1983); T. Stockamp, arXiv:hep-ph/0408206.
- [87] J. I. Kapusta, Nucl. Phys. B **148**, 461 (1979). T. Toimela, Phys. Lett. B **124**, 407 (1983). B. A. Freedman and L. D. McLerran, Phys. Rev. D **16**, 1130 (1977); D **16**, 1147 (1977); D **16**, 1169 (1977); V. Baluni, Phys. Rev. D **17**, 2092 (1978); P. Arnold and C. x. Zhai, Phys. Rev. D **51**, 1906 (1995) [arXiv:hep-ph/9410360]; C. x. Zhai and B. M. Kastening, Phys. Rev. D **52**, 7232 (1995) [arXiv:hep-ph/9507380]; K. Kajantie, M. Laine, K. Rummukainen and Y. Schroder, Phys. Rev. D **67**, 105008 (2003) [arXiv:hep-ph/0211321].
- [88] P. Danielewicz and M. Gyulassy, Phys. Rev. D **31**, 53 (1985).
- [89] D. Teaney, arXiv:nucl-th/0301099.
- [90] D. Kharzeev and K. Tuchin, Nucl. Phys. A **753**, 316 (2005) [arXiv:hep-ph/0501234].
- [91] E. V. Shuryak and I. Zahed, Phys. Rev. D **67**, 054025 (2003) [arXiv:hep-ph/0207163]; E. Shuryak, Nucl. Phys. A **715**, 289 (2003) [arXiv:hep-ph/0205031].
- [92] D. M. Ostrovsky, G. W. Carter and E. V. Shuryak, Phys. Rev. D **66**, 036004 (2002) [arXiv:hep-ph/0204224].

- [93] D. J. Gross, R. D. Pisarski and L. G. Yaffe, *Rev. Mod. Phys.* **53**, 43 (1981); R. D. Pisarski and L. G. Yaffe, *Phys. Lett. B* **97**, 110 (1980).
- [94] S. S. Adler *et al.* [PHENIX Collaboration], *Phys. Rev. Lett.* **91**, 072303 (2003) [arXiv:nucl-ex/0306021]; B. B. Back *et al.* [PHOBOS Collaboration], *Phys. Rev. Lett.* **91**, 072302 (2003) [arXiv:nucl-ex/0306025]; J. Adams *et al.* [STAR Collaboration], *Phys. Rev. Lett.* **91**, 072304 (2003) [arXiv:nucl-ex/0306024].
- [95] I. Arsene *et al.* [BRAHMS Collaboration], *Phys. Rev. Lett.* **91**, 072305 (2003) [arXiv:nucl-ex/0307003]; S. S. Adler *et al.* [PHENIX Collaboration], *Phys. Rev. C* **69**, 034910 (2004) [arXiv:nucl-ex/0308006]; K. Adcox *et al.* [PHENIX Collaboration], *Phys. Lett. B* **561**, 82 (2003) [arXiv:nucl-ex/0207009]; S. S. Adler *et al.* [PHENIX Collaboration], *Phys. Rev. Lett.* **91**, 072301 (2003) [arXiv:nucl-ex/0304022]; B. B. Back *et al.* [PHOBOS Collaboration], *Phys. Lett. B* **578**, 297 (2004) [arXiv:nucl-ex/0302015]; C. Adler *et al.* [STAR Collaboration], *Phys. Rev. Lett.* **89**, 202301 (2002) [arXiv:nucl-ex/0206011]; J. Adams *et al.* [STAR Collaboration], *Phys. Rev. Lett.* **91**, 172302 (2003) [arXiv:nucl-ex/0305015].